An Experimental Evaluation of Kalman Filtering

The effectiveness of the stationary form of the discrete Kalman filter for state estimation in noisy process systems was demonstrated by simulated and experimental tests on a pilot plant evaporator. The filter was incorporated into a multivariable, computer control system and resulted in good control despite process and/or measurement noise levels of 10%. The results were significantly better than those obtained when the Kalman filter was omitted or replaced by conventional exponential filters. In this application the standard Kalman filter was reasonably insensitive to incorrect estimates of initial conditions or noise statistics and to errors in model parameters. The filter estimates were sensitive to unmeasured process disturbances. However this sensitivity could be reduced by treating the noise covariance matrices R and Q as design parameters rather than noise statistics and selecting values which result in increased weighting of the process measurements relative to the calculated model states.

JAMES C. HAMILTON
DALE E. SEBORG
and
D. GRANT FISHER

Department of Chemical Engineering
University of Alberta
Edmonton, Alberta T6G2G6, Canada

SCOPE

Many of the design techniques based on modern control theory assume that values are available for all the state variables in the system of interest. However, in most practical situations, it is not feasible to measure all state variables and, furthermore, the measurements that are available often contain significant amounts of random noise and/or systematic errors. In these situations, on-line estimation techniques can be used to estimate the unmeasured state variables and to reduce the effects of noise. Sequential estimation techniques, or filters as they are commonly called, produce estimates of the true process values from noisy process measurements and values calculated from a suitable process model.

The Kalman filter has probably received more attention in the recent literature than any other state estimation technique. It has been applied successfully in the aerospace industry and more recently there have been a number of theoretical investigations and simulation studies of its use in process control applications. However, there have been very few reported applications to actual industrial processes.

In this investigation the effectiveness of the stationary form of the discrete Kalman filter is evaluated through simulation studies and experimental application to a pilot scale, double effect evaporator. The Kalman filter is derived using a fifth-order state space model of the evaporator and used to provide state estimates for an optimal multivariable feedback controller. The effects of design parameters, errors in the model parameters, and incorrect process statistics on the performance of the filter are examined.

CONCLUSIONS AND SIGNIFICANCE

The Kalman filter proved to be a practical and relatively easy to implement addition to the multivariable computer control system for a pilot scale, double effect evaporator. Simulation and experimental studies demonstrated that the stationary form of the discrete Kalman filter provided satisfactory state estimates even in the presence of significant noise levels, uncertain noise statistics, and significant modeling errors. However, the filter was sensitive to unmeasured step disturbances to the process.

The performance of the filter was found to depend on the accuracy of the assumed process model and the choice of the weighting matrices used in the design of the filter. The process model should, subject to reasonable complexity and model order, be as accurate as possible. Fortunately in this application the filter proved to be relatively insensitive to changes of ±25% in key model parameters. Theoretically, the weighting matrices should be made equal to the noise covariance matrices, but in practice noise statistics are not known exactly and estimates must be used. In the evaporator application the Kalman filter performed well when the matrix elements were equal to the actual noise statistics. However, it was also found that the elements of the covariance matrices could be treated as design parameters and chosen to improve the filter performance. For example, if significant modeling errors or unmeasured process disturbances are anticipated, it is desirable to make the elements of the process noise covariance matrix larger than the actual values that define the process noise statistics. This strategy leads to a filter design which places greater weighting on the process measurements and less weighting on the process model. Consequently, the effects of modeling errors and unmeasured process disturbances on filter performance are reduced.

In both the experimental and simulation studies, the use of a Kalman filter in the multivariable control scheme resulted in significantly improved control over the cases where no filter or a conventional exponential filter was used. Although not universally applicable, the results of this experimental investigation should provide useful guidelines for future applications of Kalman filtering.

Correspondence concerning this paper should be addressed to D. E. Seborg or D. G. Fisher. J. C. Hamilton is with Imperial Oil Enterprises Ltd., Calgary, Alberta.

AIChE Journal, Vol. 19, No. 5, pp. 901-909 (1973).

PREVIOUS WORK

The Kalman filter has received extensive study since publication of the classic papers by Kalman (1960) and Kalman and Bucy (1961). Despite a voluminous literature including numerous applications in the aerospace industry, it is only recently that the applicability of the Kalman filter and related nonlinear filters to chemical engineering problems has been demonstrated. However, the chemical engineering applications to date have typically consisted of simulation studies involving relatively simple process models.

The literature includes evaluations of several nonlinear filters such as the extended Kalman filter for state and parameter estimation. Simulation studies include applications to continuous stirred tank reactors by Seinfeld et al. (1969), Seinfeld (1970), and Wells (1971); to tubular and packed bed reactors by Gavalas and Seinfeld (1969), Joffe and Sargent (1972), McGreavy and Vago (1972), and Vakil et al. (1972); to heat exchangers by Coggan and Noton (1970), and Coggan and Wilson (1971b); and to a basic oxygen furnace by Wells (1970). In several of these studies, such as those by Seinfeld (1970) and Wells and Larson (1970) the filters were implemented as part of a feedback control scheme and resulted in significantly better control. The feasibility of implementing Kalman filters on small process control computers has been demonstrated by Coggan and Wilson (1971a).

Goldmann and Sargent (1971) have recently reported a detailed study of the factors affecting the performance of the Kalman filter for two simulated chemical processes. The authors considered only measurement noise in their simulations but investigated the sensitivity of the technique to errors in the design matrices, plant modeling errors, and autocorrelated measurement noise.

Only a few industrial applications of Kalman filtering have been reported in the field of process control. Åström (1970), Sastry and Vetter (1969), and Sastry et al. (1969) were concerned with applications to papermaking while Noton et al. (1968, 1970) reported the use of an extended Kalman filter in parameter and state estimation for an industrial multireactor system. Wells and Wismer (1971) have also used an extended Kalman filter to estimate the carbon content in a basic oxygen furnace.

The purpose of this investigation is to evaluate the effectiveness of a Kalman filter in a multivariable computer control scheme for a pilot scale evaporator. Of particular interest are several factors which affect the performance of the Kalman filter including poor estimates of the noise covariance matrices, unmeasured process disturbances, poor initial state estimates, and errors in the model parameters. Both simulated and experimental results are presented.

THEORY

The mathematical formulation of both the optimal control problem and the optimal estimation problem is well known and has been presented in texts and publications such as those by Bryson and Ho (1969) and Athans (1971). General reviews of filtering theory have been presented by Bucy (1970) and Rhodes (1971); of least squares theory by Swerling (1971); and of the Linear-Quadratic-Gaussian Problem by Mendel and Gieseking (1971). The following is a summary of the derivation of the discrete, stationary, standard Kalman filter used in this work.

Consider the discrete, linear, deterministic time-invariant process model in Equations (1) and (2):

$$\mathbf{x}[(k+1)T] = \mathbf{\Phi}(T)\,\mathbf{x}(kT) + \mathbf{\Delta}(T)\,\mathbf{u}(kT) + \mathbf{\Theta}(T)\,\mathbf{d}(kT) \quad (1)$$

$$\mathbf{y}(kT) = \mathbf{H}\,\mathbf{x}(kT) \quad (2)$$

In the optimal control problem, the design objective is to determine the control policy $\mathbf{u}(kT)$, $k = 0, 1, 2 \ldots N$, which minimizes a performance index such as the widely used quadratic performance index:

$$J = \mathbf{x}^T(N)\,\mathbf{S}\,\mathbf{x}(N) + \sum_{k=1}^{N-1} [\mathbf{x}^T(k)\,\mathbf{Q}_1\,\mathbf{x}(k) + \mathbf{u}^T(k-1)\,\mathbf{R}_1\,\mathbf{u}(k-1)] \quad (3)$$

where N, $\mathbf{Q}_1$, $\mathbf{R}_1$ and $\mathbf{S}$ are design parameters which must be specified a priori and $\mathbf{x}(k)$ is used to denote $\mathbf{x}(kT)$ etc. If the optimal control policy which minimizes this performance index is denoted by $\mathbf{u}^*(k)$, the optimal control law is given by

$$\mathbf{u}^*(k) = \mathbf{K}_{FB}(k)\,\mathbf{x}(k) \quad (4)$$

where the time-varying feedback control matrix $\mathbf{K}_{FB}(k)$ can be obtained from the solution of a matrix Ricatti equation (Bryson and Ho, 1969). However, an important simplification occurs if control over an infinite period of time is assumed [that is, $N \rightarrow \infty$ in Equation (3)]. In this special case, the controller matrix $\mathbf{K}_{FB}(k)$ in Equation (4) becomes a constant matrix and the optimal control law is

$$\mathbf{u}^*(k) = \mathbf{K}_{FB}\,\mathbf{x}(k) \quad (5)$$

The control law in Equation (5) is attractive for on-line computer control calculations since only a single, time invariant matrix, $\mathbf{K}_{FB}$ must be stored rather than a sequence of N time-varying matrices as in Equation (4).

In order to implement the optimal control laws defined by Equations (4) and (5), current values of all n state variables are required. If it is not feasible to measure all state variables (the usual case), then some type of state estimation technique is necessary.

In the optimal estimation problem, the objective is to calculate state estimates $\hat{\mathbf{x}}(k)$ from noisy measurements of the input and output variables such that the estimates minimize a specified performance index. It can be formulated as follows. As the stochastic process of interest, consider the deterministic process model of Equations (1) and (2) with the inclusion of random process noise $\mathbf{w}(k)$ and measurement noise $\mathbf{v}(k)$, such that

$$\mathbf{x}(k+1) = \mathbf{\Phi}\,\mathbf{x}(k) + \mathbf{\Delta}\,\mathbf{u}(k) + \mathbf{\Theta}\,\mathbf{d}(k) + \mathbf{\Gamma}\,\mathbf{w}(k) \quad (6)$$

$$\mathbf{y}(k) = \mathbf{H}\,\mathbf{x}(k) + \mathbf{v}(k) \quad (7)$$

If it is assumed that $\mathbf{v}(k)$ and $\mathbf{w}(k)$ are zero-mean, uncorrelated white-noise sequences, then the covariance matrices satisfy

$$\text{cov}\,[\mathbf{w}(k), \mathbf{w}(j)] = E[\mathbf{w}(k)\,\mathbf{w}^T(j)] = \mathbf{Q}(k)\,\delta_{kj} \quad (8)$$

$$\text{cov}\,[\mathbf{v}(k), \mathbf{v}(j)] = E[\mathbf{v}(k)\,\mathbf{v}^T(j)] = \mathbf{R}(k)\,\delta_{kj} \quad (9)$$

$$\text{cov}\,[\mathbf{w}(k), \mathbf{v}(j)] = \text{cov}\,[\mathbf{v}(k), \mathbf{w}(j)] = \mathbf{0} \quad (10)$$

where δ_{kj} is the Kronecker delta and $\mathbf{Q}(k)$ and $\mathbf{R}(k)$ are the covariance matrices for the process noise and measurement noise, respectively.

The performance specifications for a suitable state estimation algorithm (filter) include:

1. That it produce a sequential estimate of the state $\mathbf{x}(k)$ which is linear in the measurement $\mathbf{y}(k)$ and is up-

dated as each new set of measurements is obtained, and

2. The filter output $\hat{\mathbf{x}}(k)$ be a minimal variance estimate in the sense it minimizes the following performance index:

$$J = E\{[\mathbf{x}(k) - \hat{\mathbf{x}}(k)]^T [\mathbf{x}(k) - \hat{\mathbf{x}}(k)]\} \quad (11)$$

The solution to this optimal estimation problem is the Kalman filter:

$$\hat{\mathbf{x}}(k) = \overline{\mathbf{x}}(k) + \mathbf{K}(k) [\mathbf{y}(k) - \mathbf{H}\overline{\mathbf{x}}(k)] \quad (12)$$

where $\overline{\mathbf{x}}(k)$ is calculated from the deterministic model using the state estimate plus the measured process inputs from the previous time interval, that is,

$$\overline{\mathbf{x}}(k) = \mathbf{\Phi}\,\hat{\mathbf{x}}(k-1) + \mathbf{\Delta}\,\mathbf{u}(k-1) + \mathbf{\Theta}\,\mathbf{d}(k-1) \quad (13)$$

Although different in notation and form, Equations (12) and (13) are equivalent to those presented by Sage and Melsa (1971). Assuming that the process is statistically stationary (that is, Q and R are constant matrices) the gain matrix $\mathbf{K}(k)$ can be calculated from the following recursive relations:

$$\mathbf{K}(k) = \mathbf{P}(k)\,\mathbf{H}^T\mathbf{R}^{-1} = \mathbf{M}(k)\,\mathbf{H}^T\,(\mathbf{H}\,\mathbf{M}(k)\,\mathbf{H}^T + \mathbf{R})^{-1} \quad (14)$$

$$\mathbf{M}(k+1) = \mathbf{\Phi}\,\mathbf{P}(k)\,\mathbf{\Phi}^T + \mathbf{\Gamma}\,\mathbf{Q}\,\mathbf{\Gamma}^T \quad (15)$$

$$\mathbf{P}(k) = (\mathbf{I} - \mathbf{K}(k)\,\mathbf{H})\,\mathbf{M}(k) \quad (16)$$

and the assumed initial conditions:

$$\mathbf{M}(0) = \mathbf{P}(0) = E\{[\mathbf{x}(0) - \hat{\mathbf{x}}(0)]\,[\mathbf{x}(0) - \hat{\mathbf{x}}(0)]^T\} \quad (17)$$

Starting with the initial conditions in Equation (17), Equations (14) to (16) are evaluated in a sequential manner until $\mathbf{K}(k)$ converges to a constant value, K. If it is assumed that the observation time NT is long compared to the dominant time constants of the process, then the following stationary form of Equation (12) may be used:

$$\hat{\mathbf{x}}(k) = \overline{\mathbf{x}}(k) + \mathbf{K}\,[\mathbf{y}(k) - \mathbf{H}\,\overline{\mathbf{x}}(k)] \quad (18)$$

where K is the limiting solution of Equations (14) to (16) as $k \to \infty$.

There is a significant practical advantage in using the stationary version of the Kalman filter since only one matrix K need be stored instead of a large set of matrices $\{\mathbf{K}(k)\}$. A further consequence of using the stationary form of the filter is that the covariance of the error in the initial state estimate $\mathbf{P}(0)$ has no effect on the gain matrix K.

Finally, it should be noted that according to the Separation Theorem (Bryson and Ho, 1969), the optimal control policy for the stochastic system in Equations (6) and (7) consists of the optimal control law for the deterministic system [Equation (4) or (5)] with $\mathbf{x}(k)$ replaced by the optimal estimate $\hat{\mathbf{x}}(k)$ from the Kalman filter. That is,

$$\mathbf{u}^*(k) = \mathbf{K}_{FB}\,\hat{\mathbf{x}}(k) \quad (19)$$

The Separation Theorem is valid if both the process and measurement noise are Gaussian, an assumption which has not been necessary up to this point.

DESCRIPTION OF PROCESS AND PROCESS MODEL*

The pilot plant evaporator used as the subject of all the simulated and experimental investigations reported in this paper is a double-effect unit and is normally operated with a feed rate of approximately 2.27 kg/min of 3% aqueous triethylene glycol. The primary control objective is to maintain the product concentration $C2$ at a constant value of approximately 10% in spite of disturbances in the feed flow rate F, the feed concentration CF, and/or the feed temperature TF. The liquid holdups $W1$ and $W2$ must also be maintained within acceptable operating limits. The primary control, or manipulated variables, are the inlet steam flow S, the bottoms flow from the first effect $B1$, and the product flow from the second effect $B2$. The unit is heavily instrumented and can be controlled either by conventional electronic instruments or by an IBM1800 digital computer. When single variable controllers are used the normal configuration is to control C2 by manipulating S, W1 by B1 and W2 by B2. In state space terminology the control problem is to maintain the state vector $\mathbf{x}$ (or alternatively the output vector $\mathbf{y}$) equal to the desired value in spite of disturbances $\mathbf{d}$ by manipulating the control vector $\mathbf{u}$.

A fifth-order state space model of the evaporator was derived by Newell and Fisher (1972a) based on linearized material and energy balances. The discrete model equations are of the form given in Equations (1) and (2) and are expressed in terms of normalized perturbation variables (see Appendix). The definitions of the process variables and their normal steady state values are in the Notation. Of the five state variables in this model, only the first effect concentration $C1$ is not measured.

The process noise vector $\mathbf{w}$ in Equation (6) was assumed to consist of six elements corresponding to the three disturbances $\mathbf{d}$ and the three manipulated variables $\mathbf{u}$. For the simulation studies described in the next section, each element of $\mathbf{w}$ and $\mathbf{v}$ in Equations (6) and (7) was assumed to be a Gaussian noise sequence with zero mean and a standard deviation of 0.1. Thus relatively high noise levels were considered in this investigation (compare Figure 1).

SIMULATION STUDY

The effect of the following factors on the performance of the Kalman filter was determined by digital simulation:

1. Different noise covariance matrices, Q and R.
2. Unmeasured process disturbances.
3. Incorrect estimates of the initial state.
4. Errors in model parameters.

In both the simulation and experimental studies, diagonal Q and R matrices were assumed with each matrix having equal diagonal elements. That is,

$$\mathbf{Q} = q\,\mathbf{I}_s \quad \text{and} \quad \mathbf{R} = r\,\mathbf{I}_t \quad (20)$$

The physical interpretation of the Q and R matrices in Equation (20) is that the noise levels of the individual signals are identical and statistically independent. Once the ratio r/q is specified, the Kalman filter gain matrix K is uniquely determined by the asymptotic solution of Equations (14) to (16). Hence in the following discussion, this ratio rather than the absolute values of q and r will be cited. Gaussian noise sequences with standard deviations of 0.1 were used in all simulation runs, and hence the theoretically correct value of r/q was 1.0.

The effectiveness of the Kalman filter in providing estimates of the entire state vector from noisy measurements

* A schematic diagram of the evaporator is available in Newell et al. (1972 a, b, c).

is illustrated by Figure 1. The simulated responses are from the deterministic model defined by Equation (1) and the parameter matrices in the Appendix. Except for C2, which was 30% low, the initial states used for both the simulation runs and the Kalman filter derivation were the normal steady state operating conditions defined in the Notation section and by arrow heads on the vertical axis. Both process and measurement noise were added but the control and disturbance vectors were identically zero. The actual values of the state variables (which include the effect of the process noise w) are shown by the solid curves in Figure 1. The state estimates from the Kalman filter (short dashes) are very close to the actual states and represent a considerable improvement over the unfiltered data. This improvement was expected since the simulation was based on the same model used in designing the Kalman filter (that is, no modeling errors) and the noise covariance matrices were set equal to the theoretically correct values (that is, $r/q = 1$). The gain matrix for this filter is also included in the Appendix.

Effect of Using Incorrect Values for the Noise Covariance Matrices

Given accurate information concerning noise statistics, the noise covariance matrices **Q** and **R** can be set to their

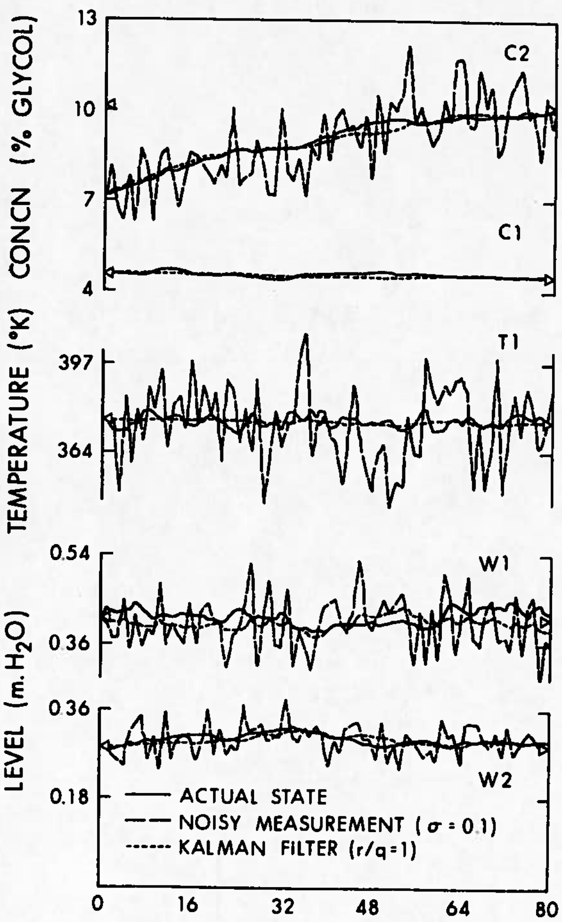

Fig. 1. Simulated open-loop responses for all the state variables of the evaporator model starting with both the actual and estimated values of the product concentration 30% below the steady state value.

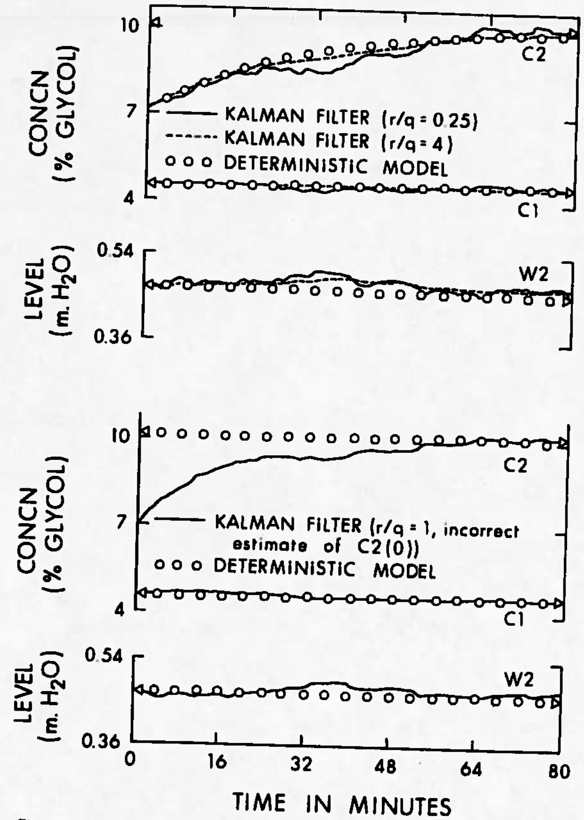

Fig. 2. Simulated open-loop evaporator responses. Data in the top half shows that the filter output becomes smoother and closer to that of the deterministic model as r/q increases. Data in the bottom half shows that errors in the filter estimate caused by incorrect estimates of the initial state are eliminated during a single transient response.

theoretically optimal values. However, accurate statistical information is rarely available in practice and it is useful to know to what extent incorrect values of **R** and **Q** affect filter performance. The consequences of using an incorrect value of r/q in designing a Kalman filter for the evaporator can be seen by comparing the filter estimates plotted in Figures 1 and the top half of Figure 2. As r/q changes from 0.25 to 4.0, the estimated response becomes closer to the deterministic model response. This follows since a large value of r/q implies relatively large measurement noise levels and small process noise levels; consequently, the resulting filter gain matrix **K** will have small elements and the estimated response will be close to the deterministic model response as is evident from Equation (18).

Effect of Poor Initial State Estimates

The lower half of Figure 2 shows the effect of an inaccurate initial state estimate on the performance of the Kalman filter. The deterministic model response was started at the correct initial state and since $\mathbf{u} = \mathbf{d} = \mathbf{0}$, the model responses remained constant. The Kalman filter provides satisfactory state estimates of the four state variables that were initialized to the correct values, and the C2 estimate gradually approaches the true value from an initial estimate of C2 which was 30% below the true value. Thus the filter estimates are reasonable after the transient due to the poor initial state estimate.

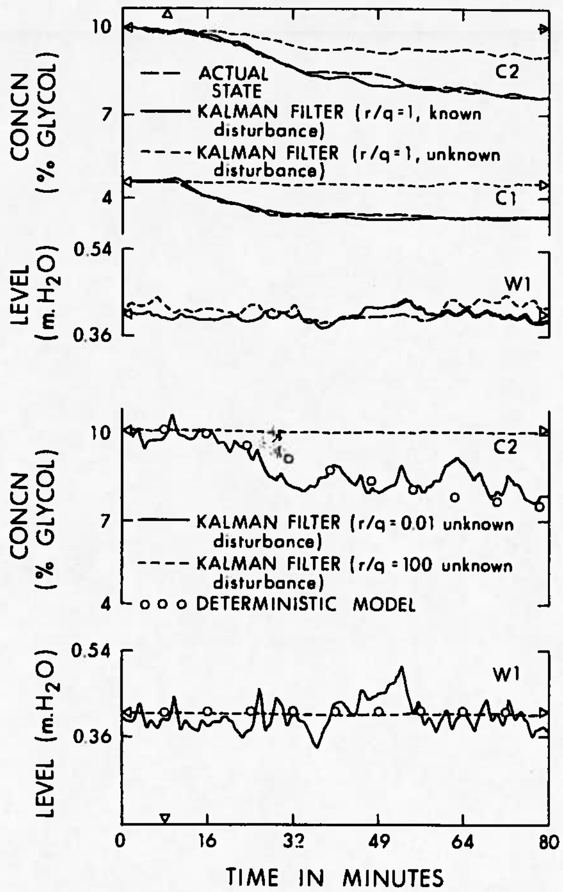

Fig. 3. Simulated open-loop evaporator responses to a −30% step change in feed composition. The data in the top half show that unknown (unmeasured) disturbances produce significant errors in the filter estimates. The data in the bottom half show that this error can be reduced by decreasing the r/q ratio (but the output is noisier).

Effect of an Unmeasured Disturbance

Measured disturbances do not normally cause any difficulty because they can be introduced into the Kalman filter as part of the disturbance vector **d** in Equation (13). Unmeasured disturbances affect the measured process outputs y but have no effect on the state $\bar{x}$, calculated from Equation (13). If **K H** in Equation (18) is equal to the identity matrix **I**, then this causes no problems. For the other extreme where $\mathbf{K} = \mathbf{0}$ the filter outputs would never reflect the influence of the unmeasured disturbance. In practical applications the errors caused by unmeasured disturbances will lie between these two limits and will be determined by the magnitude of the elements of **K**, that is, by the choice of **Q** and **R**.

The effect of an unmeasured 30% step change in feed composition is shown in Figure 3. In the top set of curves in Figure 3, the theoretically appropriate value of $r/q = 1$ is used in the filter design. The state estimates are satisfactory when the step change in feed composition is known but are significantly in error when the filter is unaware of the disturbance. The bottom set of curves in Figure 3 indicates the effect of the same disturbance when different values of r/q are used in designing the Kalman filter. For $r/q = 100$, the state estimates are very poor due to a **K** matrix with large elements. By contrast when $r/q = 0.01$, the elements of **K** are relatively small and although the state estimates are somewhat noisy, they correctly reflect trends in the state variables.

These results indicate that it may be advantageous to make the elements of **Q** artificially large (that is, r/q small) to accommodate unmeasured disturbances (and/or process nonlinearities or modeling errors).

An alternative strategy for alleviating the effects of unmeasured disturbances is to estimate the disurbances on-line. This can be achieved by augmenting the state vector with the disturbance variables and designing a Kalman filter for the augmented system (Sage and Melsa, 1971). This approach, which can also be used to deal with measurement bias and drift, is the subject of a current study by the authors.

Effect of Errors in Model Parameters

A series of simulated runs were also made to determine the effect of model accuracy on the performance of the Kalman filter. This was done by arbitrarily specifying values for parameters (such as the holdups, $W1$ and $W2$) that were 25% above or below the true values and recalculating the coefficient matrices in the state space form of the model [Equations (1) and (2)]. These new (erroneous) models were then used as a basis for calculating new filter designs, that is, new values of **K** for Equation (18). Simulated responses were then obtained by applying these different Kalman filter designs to the same (accurate) process model. In general it was found that changes of $\pm 25\%$ in $W1_{ss}$ and $W2_{ss}$ did not produce any noticeable changes in the filter estimates. Hence it was concluded that, in this application, the filter performance was not sensitive to changes in these model parameters.

Closed-Loop System

A series of closed-loop simulation runs was performed in order to evaluate the effectiveness of Kalman filters in multivariable feedback control systems. The state estimates produced by the Kalman filter were used in the multivariable control law defined by Equation (19). The optimal feedback matrix $\mathbf{K}_{FB}$ used in this work is given by Equation (21).

$$\mathbf{K}_{FB} = \begin{bmatrix} 10.78 & -1.61 & -4.82 & 0 & -19.57 \\ 5.35 & 0.36 & 0.55 & 0 & 12.49 \\ 7.52 & 1.27 & 0.18 & 24.61 & 32.69 \end{bmatrix} \tag{21}$$

The performance of the Kalman filter in this optimal control scheme when process and measurement noise occur is illustrated in Figure 4. When the control calculations are based on unfiltered noisy measurements ($\sigma = 0.1$), the closed-loop response to a 20% step change in feed flow rate contains unacceptable oscillations (compare solid curves at top). By contrast, as shown by the curves in the center of Figure 4, when the Kalman filter is designed using the correct values of the noise statistics and when the disturbance is measured, then the system response is much smoother and closer to the deterministic model response. If the feed flow disturbance is not measured and the same filter is used, satisfactory control of $C2$ results but $W1$ drifts badly (data not plotted). Fortunately, the effects of unmeasured disturbances on the filter estimates can be alleviated as discussed previously by choosing smaller values of r/q. In Figures 4 to 7 the arrow heads

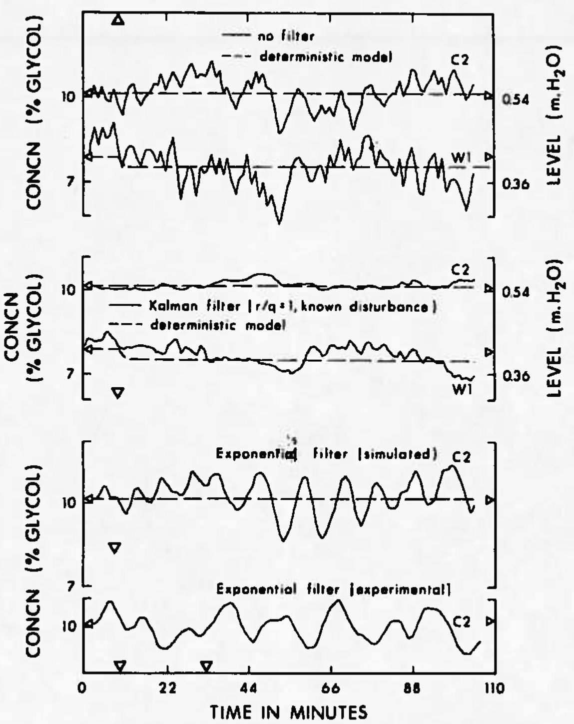

Fig. 4. Simulated closed-loop evaporator responses to a −20% step in feed flow showing that the control system incorporating a Kalman filter (centre) is better than when no filter (top) or exponential filters (bottom) are used. (The bottom curve is an experimental response to a −20% followed by a +20% disturbance.)

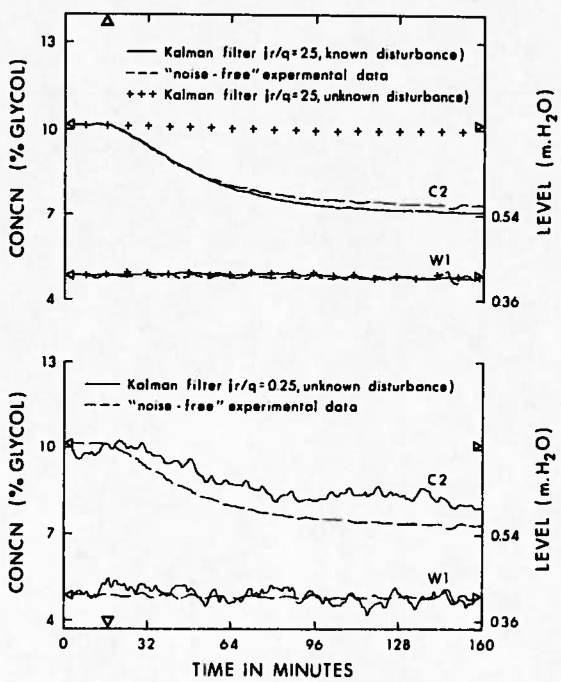

Fig. 5. Experimental open-loop responses to a −30% step change in feed composition which show the detrimental effect of unknown disturbances and confirm the effect of varying r/q observed in the simulated data of Figure 3.

on the horizontal axis denote the start of a step disturbance.

EXPERIMENTAL STUDY

The conclusions derived from the simulation studies were confirmed by experimental studies carried out on the pilot scale, double effect evaporator. Since the actual measurement and process noise levels in the evaporator are relatively low ($\sigma = 0.01$ and 0.02 respectively), zero-mean Gaussian noise with $\sigma = 0.1$ was added to the output measurements in order to provide a more severe test of the Kalman filter. The filter for the experimental study was designed using a value of $r/q = 25$ which corresponds to assumed process and measurement noise levels of $\sigma =$ 0.02 and 0.10, respectively. The gain matrix for this Kalman filter is included in the Appendix. Details concerning the implementation of multivariable computer control techniques to the evaporator have been presented elsewhere (Newell et al., 1972b, c; Hamilton, 1972).

Experimental open-loop response data for a − 30% step change in feed composition are shown, along with the corresponding filter estimates, in Figure 5. It should be noted that the experimental responses plotted in Figures 5 to 7 do not include the artificial Gaussian noise that was added to the measurements prior to the estimation and control calculations. When the Kalman filter is aware of the step disturbance, excellent state estimates result, as illustrated in the top half of Figure 5. However, when the disturbance is not measured a Kalman filter designed using $r/q = 25$ gives very poor estimates of the actual states. The estimates are improved, but noisier, when smaller r/q values are used as illustrated in the bottom half of Figure 5.

Experimental closed-loop responses with and without artificially added measurement noise are shown in Figure 6 for optimal feedback control using the gain matrix of Equation (21). In the noise-free run (bottom half), noise

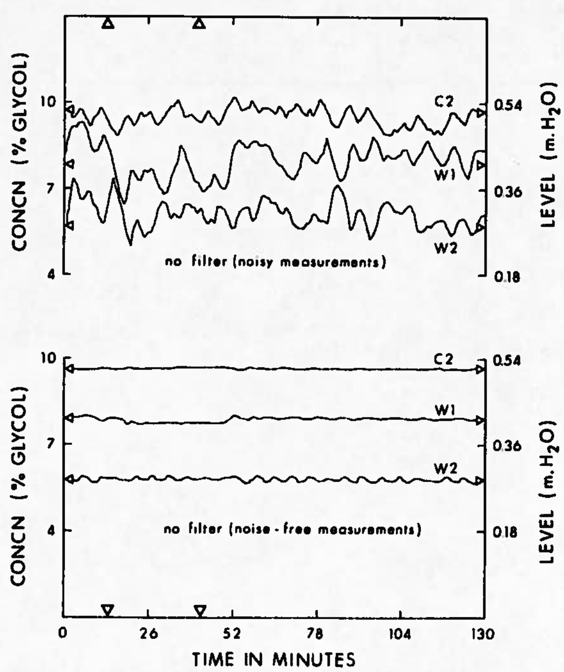

Fig. 6. Experimental closed-loop evaporator responses to 20% step changes in feed flow demonstrating the detrimental effects of increased measurement noise on the multivariable computer control-schemes (compare Figure 4).

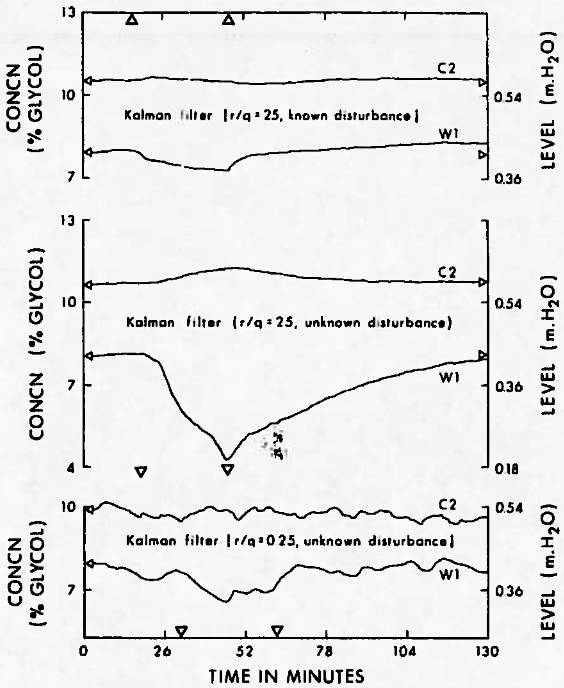

Fig. 7. Experimental closed-loop evaporator responses to 20% step changes in feed flow showing the detrimental effect of unmeasured disturbances plus the effect of varying the r/q ratio (compare Figures 5 and 3).

was not added to the measurements and excellent control was maintained in spite of two 20% feed flow disturbances. However, when unfiltered noisy measurements are used, the evaporator response exhibits excessive oscillations and is unsatisfactory as shown by the data in the top half of Figure 6. The control variables u_i are not plotted but were very noisy as would be expected.

If a Kalman filter is used, the detrimental effects of noise can be greatly reduced as illustrated in the top part of Figure 7. Here the $C2$ and $W1$ responses represent a significant improvement over the noisy responses in Figure 6 where no filter was used. However, if the filter design is based on a r/q value of 25 (which is the theoretically correct value) an unmeasured feed flow disturbance results in poor control (compare middle portion of Figure 7). A filter designed using a smaller value of r/q is less seriously affected by unmeasured disturbances. For example, as illustrated in the bottom part of Figure 7, an r/q ratio of 0.25 produces results that are a reasonable compromise between the oscillations in Figure 6 (top) and the sensitivity to disturbances in Figure 7 (center).

Thus the experimental results support the conclusions of the simulation studies concerning the effectiveness of the Kalman filter and demonstrate the utility of treating the **R** and **Q** matrices as design parameters that can be specified by the designer to tailor the process response to his specifications.

EXPONENTIAL FILTER

For comparison with the Kalman filter, simulation and experimental studies were also carried out using an exponential filter of the type commonly available in commercial Direct Digital Control (DDC) programs.

An exponential or RC filter reduces noise in a set of measurements by combining the present measurement y and the previous estimate $\hat{y}$ in a fixed proportion specified by the user. The scalar filter equation is

$$\hat{y}_i(k+1) = \hat{y}_i(k) + \alpha_i[y_i(k+1) - \hat{y}_i(k)] \quad (22)$$

Normally each filter constant α_i is assigned a value between zero and one since a value of one corresponds to no filtering and a value of zero corresponds to total filtering in which the measurements are not used. In selecting a filter constant for the exponential filter, a trade-off is involved since increasing the value of α_i increases the degree of filtering but also increases the dynamic lag introduced by the filter. It is important to realize that the exponential filter merely provides signal conditioning and does not furnish any information about unmeasured state variables.

The experimental and simulation results in Figure 4 provide a comparison of the closed-loop control resulting from the use of either a Kalman filter or an exponential filter in a multivariable control scheme. It is apparent that the Kalman filter provides better control of the primary controlled variable $C2$.

ACKNOWLEDGMENT

Financial support from the National Research Council of Canada and the assistance of the staff in the department's Data Acquisition, Control and Simulation (DACS) Centre is gratefully acknowledged.

NOTATION

$\mathbf{d}$ = disturbance vector, $p \times 1$
E = expected value
$\mathbf{H}$ = constant coefficient matrix
$\mathbf{I}_l$ = identity matrix, $l \times l$
$\mathbf{I}_s$ = identity matrix, $s \times s$
J = performance index for control problem
j = time increment counter
$\mathbf{K}$ = gain matrix for Kalman filter
$\mathbf{K}_{FB}$ = feedback control matrix
k = time increment counter
l = dimension of output vector
$\mathbf{M}$ = variance matrix
m = dimension of control vector
N = indicator of final time period
n = dimension of state vector
$\mathbf{P}$ = error covariance matrix
p = dimension of disturbance vector
$\mathbf{Q}$ = process noise weighting matrix
$\mathbf{Q}_1$ = state weighting matrix in control problem
q = process noise covariance, see Equation (20)
$\mathbf{R}$ = measurement noise weighting matrix
$\mathbf{R}_1$ = control weighting matrix in control problem
r = measurement noise covariance, see Equation (20)
$\mathbf{S}$ = final state weighting matrix in control problem
s = dimension of process noise vector
T = discrete time interval
$\mathbf{u}$ = control vector, $m \times 1$
$\mathbf{v}$ = measurement noise vector, $l \times 1$
$\mathbf{w}$ = process noise vector, $s \times 1$
$\mathbf{x}$ = state vector, $n \times 1$
$\mathbf{y}$ = output measurement vector, $l \times 1$
cov = covariance

Greek Letters

α = coefficient scalar in the exponential filter equation
$\mathbf{\Gamma}$ = coefficient matrix of the process noise

$\mathbf{\Delta}$ = control coefficient matrix
$\mathbf{\Theta}$ = disturbance coefficient matrix
σ = standard deviation
$\mathbf{\Phi}$ = transition matrix

Superscripts

$*$ = indicates optimal value of variable
T = matrix (vector) transpose
-1 = matrix inverse
$-$ = indicates model calculation
$\wedge$ = indicates estimated variable

Process Variables

State vector, $\mathbf{x}$:
$W1$ = first-effect holdup, 20.8 kg
$C1$ = first-effect concentration, 4.59% glycol
$H1$ = first-effect enthalpy, 441 kJ/kg
$W2$ = second-effect holdup, 19.0 kg
$C2$ = second-effect concentration, 10.11% glycol
Control vector, $\mathbf{u}$:
S = steam, 0.91 kg/min
$B1$ = first-effect bottoms, 1.58 kg/min
$B2$ = second-effect bottoms, 0.72 kg/min
Disturbance vector, $\mathbf{d}$:
F = feed flow rate, 2.26 kg/min
CF = feed concentration, 3.2% glycol
HF = feed enthalpy, 365 kJ/kg
Output vector, $\mathbf{y}$:
$\mathbf{y}^T$ = $[W1, H1, W2, C2]$

LITERATURE CITED

Aström, K. J., *Introduction to Stochastic Control Theory*, Academic Press, New York (1970).

Athans, M., "The Role and Use of the Stochastic Linear-Quadratic-Gaussian Problem in Control System Design," *IEEE Trans. Auto. Control, AC-16*, 529 (1971).

Bryson, A. E., Jr., and Y. C. Ho, *Applied Optimal Control*, Blaisdell, Waltham, Mass. (1969).

Bucy, R. S., "Linear and Nonlinear Filtering," *Proc. IEEE*, 58, 854 (1970).

Choquette, P., A. R. M. Noton, and C. A. G. Watson, "Remote Computer Control of an Industrial Process," *ibid.*, 10.

Coggan, G. C., and A. R. M. Noton, "Discrete-Time Sequential State and Parameter Estimation in Chemical Engineering," *Trans. Instn. Chem. Engr.*, 48, T255 (1970).

Coggan, G. C., and J. A. Wilson, "On-line State Estimation with a Small Computer," *Computer J.*, 14, 61 (1971).

———, "Approximate Models and the Elimination of Bias in On-Line Estimation of Industrial Processes," *Proc. Joint Symp. on Online Computer Methods Relevant to Chem. Eng.*, Nottingham, British Computer Society, London (1971).

Gavalas, G. R., and J. H. Seinfeld, "Sequential Estimation of States and Kinetic Parameters in Tubular Reactors with Catalyst Decay," *Chem. Eng. Sci.*, 24, 625 (1969).

Goldmann, S. E., and R. W. H. Sargent, "Applications of Linear Estimation Theory to Chemical Processes: A Feasibility Study," *Chem. Eng. Sci.*, 26, 1535 (1971).

Hamilton, J. C., "An Experimental Investigation of State Estimation in Multivariable Control Systems," M.Sc. thesis, Univ. Alberta, Edmonton, Canada (1972).

Joffe, B. L., and R. W. H. Sargent, "The Design of On-line Control Scheme for a Tubular Reactor," *Trans. Inst. Chem. Engrs.*, 50, T270 (1972).

Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," *Trans. ASME, J. Basic Eng.*, 82, 35 (1960).

———., and R. S. Bucy, "New Results in Linear Filtering and Prediction Theory," 83, 95 (1961).

McGreavy, C., and A. Vago, "Application of Nonlinear Filtering Techniques to Adaptive Optimal Control," paper presented at the 65th Ann. meeting of Am. Inst. Chem. Engrs., New York (1972).

Mendel, J. M., and D. L. Gieseking, "Bibliography on the Linear-Quadratic-Gaussian Problem," *IEEE Trans. Auto. Control*, AC-16, 847 (1971).

Newell, R. B., and D. G. Fisher, "Model Development, Reduction and Experimental Evaluation for an Evaporator," *Ind. Eng. Chem. Process Design Develop.*, 11, 213 (1972a).

———, "Experimental Evaluation of Optimal, Multivariable Regulatory Controllers with Model-Following Capabilities," *Automatica*, 8, 247 (1972b).

———, and D. E. Seborg, "Computer Control Using Optimal, Multivariable, Feedforward-Feedback Algorithms", *AIChE J.*, 18, 881 (1972c).

Noton, A. R. M., and P. Choquette, "The Application of Modern Control Theory to the Operation of an Industrial Process," *Proc. IFAC/IFIP Symp.*, Toronto, Ontario (1968).

Rhodes, I. B., "A Tutorial Introduction to Estimation and Filtering," *Proc. 1969 Nat. Conf. Auto. Control*, Edmonton,

Sage, A. P., and J. L. Melsa, *Estimation Theory with Applications to Communications and Control*, McGraw-Hill, N. Y. (1971).

———, and R. Caston, "The Application of Identification Methods to an Industrial Process," *1969 JACC Preprints*, p. 787, Boulder, Colorado (1969).

Sastry, V. A. and W. J. Vetter, "A Papermaking Wet-end Dynamics Model and Parameter Identification by Iterative Filtering," *Proc. 1969 Nat. Conf. Auto. Control*, Edmonton, Canada (1969).

Seinfeld, J. H., "Optimal Stochastic Control of Nonlinear Systems," *AIChE J.*, 16, 1016 (1970).

———, G. R. Gavalas, and M. Hwang, "Control of Nonlinear Stochastic Systems," *Ind. Eng. Chem. Fundamentals*, 8, 257 (1969).

Swerling, P., "Modern State Estimation Methods from the Viewpoint of the Method of Least Squares," *IEEE Trans. Auto. Control*, AC-16, 707 (1971).

Vakil, H., M. L. Michelsen, and A. S. Foss, "Fixed Bed Reactor Control with State Estimation," Paper presented at 65th Annual AIChE Meeting, New York (1972).

Wells, C. H., "Application of Modern Estimation and Identification Techniques to Chemical Processes," *AIChE J.*, 17, 966 (1971).

———., "Optimum Estimation of Carbon and Temperature in a Simulated BOF," *1970 JACC Preprints*, p. 7, Atlanta, Georgia (1970).

———., and R. E. Larson, "Application of Combined Optimum Control and Estimation Theory to DDC," *Proc. IEEE*, 58, 16 (1970).

Wells, C. H., and D. A. Wismer, "Advances in Process Control Applications," in *Advances in Control Systems*, Vol. 8, C. T. Leondes (ed.), Academic Press, N. Y. (1971).

APPENDIX

State Space Evaporator Model

The model is of the form of Equations (1) and (2) with the elements of x, u, d and y defined as normalized perturbation variables, for example,

$$x_1 = \frac{W1 - W1_{ss}}{W1_{ss}}$$

where $W1_{ss}$ is the normal steady state of $W1$. Previous evaporator studies by Newell and Fisher (1972b) have shown that a discrete time interval of $T = 64$ seconds gives satisfactory control. The coefficient matrices for $T = 64$ seconds are

$$\mathbf{\Phi} = \begin{bmatrix} 1.0 & -0.0008 & -0.0912 & 0 & 0 \\ 0 & 0.9223 & 0.0871 & 0 & 0 \\ 0 & -0.0042 & 0.4377 & 0 & 0 \\ 0 & -0.0009 & -0.1052 & 1.0 & 0.0001 \\ 0 & 0.0391 & 0.1048 & 0 & 0.9603 \end{bmatrix}$$

$$\mathbf{\Delta} = \begin{bmatrix} -0.0119 & -0.0817 & 0 \\ 0.0116 & 0 & 0 \\ 0.1569 & 0 & 0 \\ -0.0137 & 0.0847 & -0.0406 \\ 0.0137 & -0.0432 & 0 \end{bmatrix}$$

$$\mathbf{\Theta} = \begin{bmatrix} 0.1182 & 0 & -0.0050 \\ -0.0351 & 0.0785 & 0.0049 \\ -0.0135 & -0.0002 & 0.0662 \\ 0.0012 & 0 & -0.0058 \\ -0.0019 & 0.0016 & 0.0058 \end{bmatrix}$$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Kalman Filter Gain Matrices

for $r/q = 1$:

$$\mathbf{K} = \frac{1}{1000} \begin{bmatrix} 137. & -6.83 & -18.3 & -0.554 \\ -24.4 & 5.26 & -7.76 & 20.8 \\ -6.83 & 34.4 & -5.76 & 5.98 \\ -18.3 & -5.76 & 90.3 & -31.3 \\ -0.554 & 5.98 & -31.3 & 36.7 \end{bmatrix}$$

For $r/q = 25$:

$$\mathbf{K} = \frac{1}{1000} \begin{bmatrix} 29.0 & -0.339 & -1.67 & 0.422 \\ -2.17 & 0.248 & -0.639 & 1.41 \\ -0.339 & 1.44 & -0.284 & 0.284 \\ -1.67 & -0.284 & 20.4 & -2.48 \\ 0.422 & 0.284 & -2.48 & 3.87 \end{bmatrix}$$

Manuscript received October 10, 1972; revision received March 20 and accepted April 5, 1973.